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Implicit Analogies in Learning: Supporting Transfer by Warming Up

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Abstract
Analogies between old and new concepts are common during classroom instruction. Previous transfer studies focused on how features of initial learning guide later, spontaneous transfer to new problem solving. We argue for a shift in the focus of analogical-transfer research toward understanding how to best support analogical transfer from previous learning when children are engaged in new learning episodes. Such research may have important implications for teaching and learning in mathematics, which relies heavily on analogies between old and new information. Some existing research promotes supporting explicit connections across old and new information within an analogy. Alternatively, we argue that teachers can invite implicit analogical reasoning through warm-up activities designed to activate relevant prior knowledge. Warm-up activities close the transfer space between old and new learning without additional direct instruction.

Keywords
analogy, transfer, mathematics learning

In classrooms, teachers often have opportunities to leverage learners' rich, prior knowledge to improve understanding of new concepts. This is especially true in mathematics, in which early formal classroom learning predicts later classroom learning, over and above individual differences in variables such as IQ (Siegler et al., 2012). Laboratory-based studies on children's math learning provide evidence for analogical learning; children demonstrate deeper understanding of novel concepts when an analogy is drawn across earlier- and later-learned concepts (e.g., Sidney & Alibali, 2015; Thompson & Opfer, 2010). In practice, mathematics standards adopted widely across the United States directly indicate that teachers should draw on children's earlier-mastered concepts (see Fig. 1). Although these recommendations are in line with theories of analogy (Gentner, 1983) and analogical learning (Gick & Holyoak, 1980), less empirical research has focused on factors that may govern robust classroom learning from distant analogies made across developmental time.

Analogical Learning Is Transfer

Analogical learning is, critically, an issue of transfer. In an instructional analogy, when a teacher makes a connection between a more familiar topic in mathematics and a novel or more complex one, the goal is often to shape children's understanding of the mathematical relationships in the novel topic on the basis of the children's prior knowledge of the familiar topic. In this view, learning via analogy involves transfer of familiar ideas from topic A (the transfer *source*) to related topic B (the transfer *target*) when both topics share the same underlying conceptual structure (see Fig. 1). Although there is a vast literature on transfer of knowledge (e.g., Barnett & Ceci, 2002; Gick & Holyoak, 1980; Klahr & Chen, 2011; Thorndike, 1913), much of the existing research has a limited application to understanding how to make such connections during new instruction.

Most empirical cognitive-science research on transfer has addressed the question of how we can teach today in a way that helps learners apply this knowledge tomorrow. To successfully transfer learning from topic A to later problem solving, solvers must possess a robust, abstract, and portable representation of the

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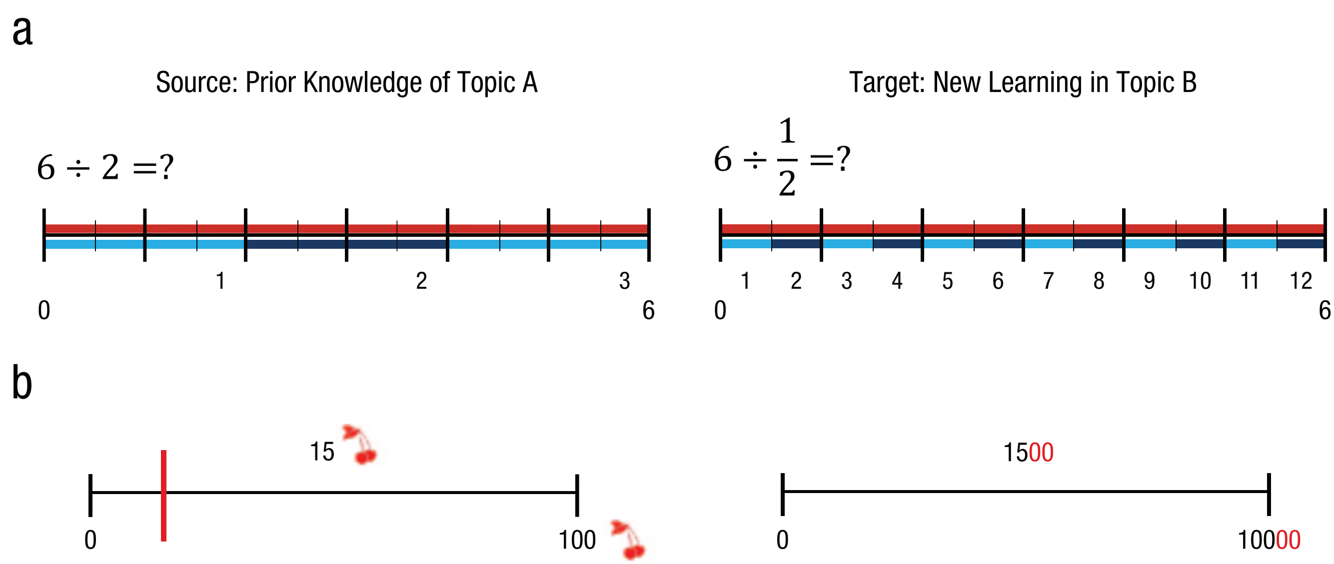

Fig. 1. Two examples of distant analogies across developmental time in mathematics. The example in (a) illustrates an analogy between whole-number division (topic A) and fraction division (topic B; e.g., see Sidney, Thompson, & Rivera, 2019). Common standards in mathematics practice suggest that one should rely on children's knowledge of whole-number division when teaching them about fraction division. This is because in each case, division can be represented as partitioning the first number into segments as long as the second number. If this analogy is indeed helpful, children with higher-quality and more abstract whole-number division knowledge would be better prepared to transfer their knowledge to fraction division. But as teachers are planning their fraction-division lesson, how should such transfer be invited and supported by the fraction-division learning activities? The example in (b) illustrates an analogy between numerical magnitudes between 0 and 100, a range in which second graders make precise estimates, and magnitudes between 0 and 10,000, a range in which second graders do not make precise estimates (see Thompson & Opfer, 2010). Just as 15 is located 15% of the way between 0 and 100, 1,500 is located 15% of the way between 0 and 10,000, but how should children be invited to draw on their relevant prior knowledge of smaller scales to bootstrap their understanding of larger scales?

underlying structure of topic A (Kaminski, Sloutsky, & Heckler, 2008). Many features of instruction, such as comparing multiple examples of the same concept during initial learning (Gentner, Loewenstein, & Thompson, 2003), fading from more concrete to more abstract examples (McNeil & Fyfe, 2012), and using common labels to highlight abstract relationships across structurally similar problems (Gentner & Hoyos, 2017), can highlight the relational structure in a learning episode and lead to increased transfer to later problem solving.

Although this research on developing portable representations informs instruction about the source that has a higher likelihood of transferring to later problem solving, it is less informative for understanding how instructors could invite transfer from prior knowledge during target learning (Schwartz, Bransford, & Sears, 2005; Schwartz & Nasir, 2003). Although many studies of analogical transfer have examined transfer from one newly learned problem to another (e.g., Gick & Holyoak, 1980), children come to the classroom with substantial prior experience, and little is known about how to leverage these highly familiar concepts. How can theories of transfer inform how we help learners draw on their past experiences to more easily make sense of today's instruction? How does viewing this practical issue through a theoretical lens reveal new theoretical questions about the nature of transfer and analogical processing? These are the key questions we aim to address in our review.

Leveraging Children's Prior Knowledge

There are many potential ways of inviting successful analogical transfer from prior knowledge during target instruction; however, little research has directly addressed this. In the following sections, we highlight two current theories of analogy and transfer and discuss their implications for making analogies during new instruction. First, we introduce a traditional view of transfer as infrequent, effortful, and difficult to achieve, which suggests that analogical instruction should occur explicitly, drawing learners' direct attention to connections between old and new concepts. Second, we introduce a newer, emerging view of transfer as ubiquitous, spontaneous, and fundamentally grounded in perception and action, which suggests that analogical instruction can occur implicitly, inviting learners to rely on their prior knowledge without directing their attention to the analogy. Detailing transfer and analogy research with both children and adults, we present an argument

for the unique benefits of implicit analogical instruction and describe how implicit analogical instruction can be instantiated by inserting brief warm-up activities immediately prior to target instruction on challenging mathematical ideas.

Explicit support for analogy

Informed by decades of research on analogical transfer to enhance later problem solving (e.g., Gick & Holyoak, 1980), some researchers have pointed toward the effectiveness of providing explicit support to help learners use what they know. In this view, transfer across an analogy is effortful and unlikely to occur without prompting. Thus, the primary challenges of linking old and new ideas are (a) identifying which aspects of prior knowledge are most helpful for new learning and (b) mapping the related elements and underlying structure across the two topics (e.g., Richland, Zur, & Holyoak, 2007; Vendetti, Matlen, Richland, & Bunge, 2015). Indeed, teachers in the United States often make explicit analogies, guiding students' identification of related source topics (Richland et al., 2007). For example, imagine a science lesson in which students are learning about atomic structure and their teacher makes an analogy to the solar system, which guides students' identification of the solar system as a relevant aspect of their prior knowledge and as a good candidate for transfer. However, Richland and colleagues (2007) observed that U.S. teachers provide little support for mapping; for example, they fail to explain which properties of the solar system correspond with which properties of atomic structure. Without mapping support, it may be difficult for students to discern which aspects of topic A should be transferred to topic B, leading to incorrect inferences (Sidney & Alibali, 2015). For instance, in our atomic-structure example, a learner might incorrectly infer that electrons, like planets, have stable orbits.

To alleviate the burdens of identifying and mapping the analogy, researchers have proposed a straightforward solution: Directly instruct learners on the analogy (Richland et al., 2007). Explicit hints or instruction to draw on previously learned problems from topic A support identification (e.g., Gick & Holyoak, 1980). Furthermore, presenting topics A and B with side-by-side visual representations and explicitly instructing students about their similarities and differences enhances students' learning from comparison (Richland & McDonough, 2010; see also Vendetti et al., 2015). These recommendations are well aligned with other research on classroom learning (e.g., Klahr & Nigam, 2004); learners are unlikely to discover important structural relationships without direct instruction that highlights the most important information (Kirschner, Sweller, & Clark, 2006). From this perspective, optimal analogical instruction should explicitly draw learners' attention to the analogy via direct instruction.

Unlike in many studies of transfer, however, analogies used during classroom instruction already include direct instruction on the target. For example, in an explicit analogy between familiar topic A (e.g., whole-number division) and novel topic B (e.g., fraction division) during classroom instruction, a teacher must remind children of topic A, instruct them on topic B, and explicitly instruct them on the relationships between topic A and topic B. Thus, we raise a criticism of the explicit approach: Direct instruction on an analogy, in addition to target instruction, may be an overwhelming amount of information for young learners to process (Sweller, van Merrienboer, & Paas, 1998). Requiring the learner to attend not only to topic B but also to topic A as well as each mapping between topics A and B imposes a high working memory load (cf. Simms, Frausel, & Richland, 2018). Richland and colleagues (Richland & McDonough, 2010; Richland et al., 2007; Vendetti et al., 2015) suggest that the strategies that support mapping (e.g., presenting visuals of topics A and B side by side) reduce learners' working memory load when they are comparing multiple problems. However, it remains an open question whether these instructional strategies compensate for the added load of explicitly attending to each component of an analogy.

Implicit support for analogy

If direct instruction on an analogy presents new challenges for learners, what might be an effective alternative for leveraging learners' prior knowledge? Given that some amount of direct instruction appears necessary, a more fruitful goal for research on analogies in instruction is understanding how analogies to students' prior knowledge can guide interpretation of new instruction rather than focusing on how to support students' effortful application of old ideas to new instruction. This view of leveraging children's prior knowledge is informed by emerging theories of transfer in which transfer is conceptualized as a spontaneous, and perhaps even implicit, process.

In their cross-disciplinary review of transfer research, Day and Goldstone (2012) argued that adult learners' prior knowledge guides their interpretation of new situations by affecting their perception of those situations and thus shapes learners' construction of mental models without requiring their conscious awareness of this process. In a series of innovative studies, researchers (e.g., Day & Gentner, 2007; Day & Goldstone, 2011; Schunn & Dunbar, 1996) have demonstrated that when adults must make sense of a new, ambiguous situation, their

recently activated knowledge from a preceding task affects their understanding of the new situation. Interestingly, when asked directly, some adults even denied that prior knowledge resolved ambiguity (Day & Gentner, 2007). This perspective may be useful for considering how instructors can implicitly support children's transfer across analogical topics without directly instructing them on the analogical connections.

Here, we refer to implicit support for analogical transfer without direct instruction as an *implicit analogy*. Implicit analogies are analogies that are supported by intentional structure in the learning environment but to which learners' attention is not explicitly drawn. Because learners' attention is not explicitly drawn to the connections across topics, we argue that this version of analogical instruction does not impose the same working memory load as explicit analogy because it does not depend on learners' ability to attend to connections or to actively maintain a mental representation of prior concepts during new learning.

In our own work (Sidney & Alibali, 2015, 2017; Sidney, Thompson, & Rivera, 2019; Thompson & Opfer, 2010), we have demonstrated that children engage in more accurate problem solving and efficient learning in challenging domains when their highly practiced, relevant prior knowledge has been recently activated using warm-up activities designed to instantiate an implicit analogy. For example, we demonstrated that children's conceptual understanding of fraction division was more likely to be accurate when structurally similar whole-number division concepts were recently practiced (Sidney & Alibali, 2017). When children had just modeled whole-number division problems using physical objects to show division by partitioning a given amount into groups, they were more likely to accurately model fraction division compared with children who modeled related but structurally dissimilar fraction multiplication immediately before fraction division. Importantly, children's attention was never drawn to the analogical similarities between problems, and children rarely explicitly commented on making analogical connections during their verbal explanations, suggesting that they may have been unaware of the implicit analogy that enhanced their problem solving.

In another study (Sidney & Alibali, 2015), we demonstrated that children can benefit from classroomlike implicit analogies during instruction: Children who practiced with a structurally similar topic A concept gained a greater understanding of the conceptual structure of topic B problems following direct instruction on topic B compared with children who had practiced with other structurally dissimilar math problems. Perhaps surprisingly, the benefit of warming up was greater when children's attention was not drawn to the analogy than when it was, suggesting an advantage for implicit analogies in instruction over explicit cues to engage in analogical learning.

Together, these findings suggest that children's prior knowledge can structure their perception of new learning materials when their relevant mental models from topic A have been recently activated. This idea is consistent with children's learning that demonstrates spontaneous transfer to more complex problems (e.g., Fyfe, McNeil, & Borjas, 2015; Gentner et al., 2016; Klahr & Nigam, 2004) and analogical transfer with infants (e.g., Chen, Sanchez, & Campbell, 1997). Even 13-month-old children spontaneously apply recently learned strategies (e.g., pulling a toy by a string) to new problems with different features without any explicit hints or instruction from an adult illustrating how to map between old and new problems.

However, the manner in which children's prior knowledge is activated affects the likelihood of transfer across such implicit analogies. Recently, we demonstrated that after warming up with whole-number division, children's conceptual models for fraction division were particularly robust when both division tasks were visually represented on a number line, as compared with being visually represented with circles, rectangles, or not at all (Sidney et al., 2019). This may occur in part because children's strategies for representing the division relationship are identical in the number-line context (e.g., partitioning a longer length into several smaller lengths; see Fig. 1) but slightly different in the other contexts (e.g., grouping whole circles in whole-number division vs. partitioning circles in fraction division). In another domain of mathematics, we reported that an implicit analogy made from young children's numerical-magnitude understanding of small, more familiar numbers (0–100) can shape their understanding of numerical magnitudes in larger ranges (e.g., 0–1,000, 0–10,000, 0–100,000; Thompson & Opfer, 2010). Children benefited most from the analogy when each unfamiliar trial was spatially aligned with a perceptually similar familiar trial. We argue that because of strategy alignment, spatial alignment, and perceptual similarities across warm-up and target problems, direct instruction on the link was not necessary for robust, spontaneous transfer between familiar and unfamiliar problems.

Open questions about supporting analogy

This emerging evidence does suggest that even children's prior knowledge can guide their construal of abstract relationships (e.g., Day & Goldstone, 2012) without explicitly directing their attention to the analogy. However, there are many open questions about how to best support implicit analogy—questions that can also help

refine our understanding of when spontaneous transfer occurs and why. On the basis of our work and others' research on transfer (e.g., Barnett & Ceci, 2002; Klahr & Chen, 2011), we can make some preliminary hypotheses about how implicit analogy may be best instantiated through warm-up activities. First, we hypothesize that warm-up activities may help to leverage children's prior knowledge by closing the *transfer distance* (Klahr & Chen, 2011), making spontaneous transfer more likely to occur. If that is the case, warm-up activities should be most effective when they activate children's relevant prior knowledge (a) very close in time to direct instruction (e.g., transfer across minutes is more likely than transfer across years), (b) with tasks that are similar to those used in instruction (e.g., transfer from one hands-on activity to another is more likely than transfer from a problem-solving worksheet to a hands-on activity), and (c) in the same context as instruction (e.g., transfer within school contexts is more likely than transfer across school and home). Importantly, because warm-up activities tend to reactivate very familiar prior knowledge, the specific characteristics of examples used in the initial learning context of that prior knowledge should be much less important than the characteristics of the warm-up activity that serves to reactivate prior knowledge. Finally, in line with Day and Goldstone's (2012) conceptualization of transfer, we also suggest that warm-up activities may be especially powerful when they involve spatial representations (e.g., Thompson & Opfer, 2010) and physical or imagined action (e.g., Sidney & Alibali, 2017). Future work is needed to test these hypotheses for implementing implicit analogical instruction and the circumstances under which implicit analogy might (or might not) be effectively activated within and outside of the domain of math.

Furthermore, the relative benefits of implicit and explicit analogy remain unclear. Implicit analogy may be most effective when domains are highly familiar and well practiced, as in the whole-number examples presented here (see Fig. 1). In contrast, explicitly drawing students' attention to similarities and differences may be necessary before they can draw connections between novel topics and learned topics that are less familiar and thus are less likely to be activated in a warm-up.

Conclusions and Future Directions

We find much promise in these initial investigations of implicit analogy as a way of supporting transfer to leverage children's prior knowledge, and there are many potential future directions for research in this area. First, we see implicit analogies as being related to other ways of inviting children to think deeply about concepts in familiar contexts before learning about them in novel or abstract contexts, for example, through inventing activities (Schwartz, Chase, Oppezzo, & Chin, 2011) or through fading from more concrete to more abstract examples (Fyfe & Nathan, 2019; McNeil & Fyfe, 2012). However, it remains to be seen whether a single shared mechanism (e.g., priming of familiar conceptual relationships) underlies the effects of more concrete preparatory activities, such as our proposed warm-ups, on subsequent instruction. Second, little is known about the role of individual differences in learning from explicit and implicit analogies. Explicit and implicit analogies may have different implications for children who engage more readily in relational reasoning and have greater resources for inhibiting irrelevant information. For example, there may be children who spontaneously seek to draw analogies across similar concepts, and the nature of the analogical instruction may not be of much consequence for their learning. In contrast, children who have difficulty mapping across an explicit analogy or inhibiting attention to misleading perceptual similarities or differences across an analogy may benefit from an implicit analogy that does not rely on explicit attention to such connections.

Finally, we highlight a major practical challenge: how to determine the most useful analogies. Our work has focused primarily on mathematics, in part because many mathematics concepts are highly complex, and helping learners make use of their prior knowledge of relevant conceptual relationships through analogy may help reduce this complexity. Furthermore, math is fundamentally a system of relationships—even numbers themselves are relational (Opfer & Siegler, 2007; Sidney, Thompson, Matthews, & Hubbard, 2017; Thompson & Opfer, 2010)—and thus invites analogical learning. However, to use analogies, teachers must have sufficient content knowledge (see Ball, Hill, & Bass, 2005) to appreciate fundamental connections between old and new concepts. If teachers are unaware of these connections, it would be impossible to foster analogical learning of any type, implicit or explicit. One strategy for implementing implicit analogies is to incorporate relevant warm-up activities in textbook materials, though teachers would still need pedagogical knowledge about when to introduce these activities on the basis of the timing of target instruction. As these challenges for future work show, empirical investigations of how children effectively leverage their prior knowledge during new learning episodes are critically important for understanding the nature of children's analogical reasoning and for developing new teaching strategies that can help children make connections in math across distal points in developmental time.

Recommended Reading

Day, S. B., & Goldstone, R. L. (2012). (See References). A comprehensive review article that fully articulates the view of transfer that motivates our conceptualization of implicit analogy.

Gentner, D. (1983). (See References). A classic article presenting the theoretical analysis of analogy that underlies most research on analogical learning and transfer.

Klahr, D., & Chen, Z. (2011). (See References). Provides a more complete discussion of transfer in general, as applicable to school contexts.

Vendetti, M. S., Matlen, B. J., Richland, L. E., & Bunge, S. A. (2015). (See References). An accessible review article that provides an overview of studies of analogical reasoning that could apply to classroom learning and presents a case for explicit analogical support.

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